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Lord Krishna and Space-time near the Event horizon in the Extreme Case

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***Abstract**: Lord Krishna says" I Am Time ". Then came words Day, Month and Year. Then three orbits of celestial object as Radius of marginally stable orbit(r m s) , Radius of Marginally bound orbit (r m b)and Radius of photon orbit(r p h) are calculated for the direct and retrograde motions. In various orbits near the event horizon are considered, the components of velocity vector of the test particle and the tangent vector field to the photon trajectories are evaluated. Minimum energy for untapped orbits is associated with orbit out of the plane. If the particle is not trapped by a black hole it is possible that gravitational radiation from it may cause to relax a part of its energy orbit. In case of Schwarzschild metric also, he obtained the three orbits as rms = 6m, rms = 4m and rph = 3m. But in this case there are no circular orbits corresponding to r = rms, r = rmb and r = rph, coinciding with the event horizon*

INDRODUCTION:

Let us suppose such a stage when a particle is not trapped by a black hole, it is possible that gravitational radiation from it may cause to relax a part of its energy orbit. In view of the great importance of circular orbits in astrophysical calculations only such orbits will be considered here. Certain types of particles orbits out of the equatorial plane have been considered by Welkins. Circular orbits in the equatorial plane of black hole have been examined by Bardeen, Press and teukolskey. A detailed study of the properties of time-like and null geodesics, especially of the stable circular orbits, in the charge free Kerr metric have been presented by Bardeen. He had found that the radius for marginally stable circular orbit, the radius for marginally bound circular orbit and the radius for circular photon orbit for both direct and retrograde motion in the case of extreme Kerr metric and also of Schwarzschild metric. In his case in three direct orbits, mentioned above, all seem to coincide with the event horizon in the extreme Kerr black hole. This apparent conflict with the null character of the horizon is because of the fact that the radial co-ordinate 'r' misrepresents the geometry of the space-time near the event horizon in the extreme case. An infinitesimal range 'r' near the horizon can correspond to an infinite range of proper radial distance. He has obtained the three different orbits for r = rms(marginally stable), r = rmb(marginally bound) and r = rph(photon orbit) near the horizon by a suitable approximation. In case of Schwarzschild metric also, he was obtained the three orbits as rms = 6m, rms = 4m and rph = 3m. But in this case there are no circular orbits corresponding to r = rms, r = rmb and r = rph, coinciding with the event horizon.

It has been shown that in addition to the stable circular orbits obtained by Bardeen there exists a circular orbit outside the ergo sphere of the Kerr black hole where also the direct marginally stable, the marginally bound and circular photon orbits coincide.

In the Kerr metric, if the rotation of the central body become slower and slower, its specific angular momentum tends to become zero and in that case the Kerr metric in limit tends to become the Schwarzschild metric. In such a limiting case, the orbits corresponding to r m s, r m b and r p h near the horizon have also been obtained

Energy, angular momentum and other physically interesting quantities are obtained for circular orbits in Equatorial plane of charge free black hole are obtained.

As, rms, rmb and rph are calculated for the direct and retrograde motions. In various orbits near the event horizon are considered, the components of velocity vector of the test particle and the tangent vector field to the photon trajectories are evaluated the frequency shift of emitted radiation from various circular orbits are considered.

BOYER-LINDQUIST OF CO-ORDINATES:

The charge free Kerr metric in usual Boyer-Lindquist of co-ordinates is -

$$ds^2 = \rho^2\Delta^{-1}dr^2 + \rho^2 d\theta^2 + \rho^{-2}Sin^2\theta\ [adt - (r^2 + a^2)d\varphi]^2 - (r^{-2}\Delta\ [dt - aSin^2\theta\, d\varphi]^2$$

$$= Adr^2 + Bd^2\theta + Cd\varphi^2 - Ddt^2 - 2Fd\varphi dt$$

where,

$$\rho^2 = r^2 + a^2Cos^2\theta,$$

$$A = \rho^2\Delta^{-1}, \qquad B = \rho^2,$$

$$C = \rho^{-2}Sin^2\theta\,[(r^2 + a^2)^2 - \Delta a^2 Sin^2\theta],$$

$$D = \rho^{-2}\,[\Delta - a^2Sin^2\theta],$$

$$F = 2mraSin^2\theta\ \rho^{-1/2},$$

$$\Delta = r^2 - 2mr + a^2,$$

a, m are respectively, the specific angular momentum and mass of the black hole.

The equations of motion in this case have been given by Carter (41) as follows:

$$\rho^2 r. = \pm\sqrt{R},$$

$$\rho^2\theta. = \pm\sqrt{q},$$

$$\rho^2\phi. = [LSin^{-2}\theta - aE] + a\Delta^{-1}P,$$

$$\rho^2 t. = a[L-aESin^2\theta] + (r^2 + a^2)\Delta^{-1}P,$$

$$\theta = Q - Cos^2\theta\,[a^2(\mu^2 - E^2) + L^2Sin^{-2}\theta],$$

$$P = E(r^2 + a^2) - La,$$

$$R = P^2 - \Delta[\mu^2 r^2 + Q\ (L - aE^2)],$$

the dots denote differentiation with respect to a parameter λ, defined in terms of the proper time by $\tau = \mu\lambda$.

The signs in the above equations can be chosen independently. E, L and Q are three constants of the particle's motion. E and L refer respectively, to the energy and to the ϕ -component of angular momentum, Q is related to the θ velocity, θ.

Wilkins has shown that an orbit has Q = 0 if and only id is confirmed to the equatorial plane (θ = π/2) of the Kerr metric. This gives from above

$$R = E^2 r^2 (r^2 + a^2) + 2mr(L - aE)^2 - r^2(L^2 - \Delta\mu^2) \quad .... \quad (10)$$

For the stable circular orbits the particles radial co-ordinate will be stable at some value of 'r' if R(r) vanishes for that values of 'r' and become negative nearby. This will be the case if,

$$R(r) = 0$$

$$\frac{\partial R}{\partial r} = 0$$

$$\frac{\partial^2 R}{\partial r^2} = 0$$

If instead of the result obtained we have $\partial^2 R/\partial r^2 > 0$, the orbit will be unstable circular orbit.

The effective radial potential is defined as that value of E which annuls the expression. In a circular orbit, the test particle experiences a minimum effective potential.

By solving above pair of equations for E and L as functions of 'r' we get for the circular orbits following expressions:-

$$\frac{E}{\mu} = \sqrt{\frac{2r(r - m)}{2r^2 - k^2(r^2 + a^2)}}$$

$$L \qquad \Delta^{½}[a\ 2m + k\ \ r(r^2 + a^2)]$$

$$\frac{}{\mu} = \pm \frac{}{\sqrt{(K^2+1)2m(r^2+a^2) - \left[a\sqrt{2m} + K\sqrt{r(r^2+a^2)}\right]^2}}$$

where,

$$K = \frac{ar(r-m)\sqrt{2mr} + k\sqrt{2rm\Delta}}{\sqrt{(r^2+a^2)\left[r(r-m)(2m-r) + m\Delta\right]}}$$

And the upper sign is for direct orbit (L>0) and lower sign is for retrograde orbit (L<0).

Where as other physically interesting quantities for the circular orbits are the angular of the particle as seen from infinity and the linear velocity of orbit relative to locally non-rotating observer. They are given respectively as,

$$\Omega = \frac{d\varphi}{= dt} \quad \frac{a\sqrt{2mr} + (r-2m)k(r^2+a^2)}{\sqrt{2mr}(r^2+a^2) - 2amk\sqrt{(r^2+a^2)}}$$

$$\upsilon\varphi = \frac{\Delta^{1/2}\left[a\sqrt{2mr} + kr\sqrt{(r^2+a^2)}\right]}{\left[\sqrt{2mr}(r^2+a^2) - 2amk\sqrt{(r^2+a^2)}\right]}$$

Configuration of energy levels of celestial bodies:

At large r(r >– m) both direct and retrograde orbits are bound having nearly equal binding energies. At small 'r', due to certain "spin-orbit coupling" affect, the binding energy increases for the direct orbit and decreases for the retrograde orbit. The direct equatorial orbit has the maximum binding energy (1 - E/μ) (and the maximum value of L/μ). The counter revolving equatorial orbit has the smallest binding energy. The maximum binding energy is reached at the radius where the orbit becomes unstable. Hence, the bound orbit. Such an orbit is situated at the inflexion point of the effective radial potential that is,

$$\text{with} \quad \frac{\partial 2R}{\partial r2} = 0$$

For such an orbit, we find from the following:

$$K^2 = \frac{2r^2m}{(r^2 + a^2)(3r - 2m),}$$

$$[ar(r - m)\sqrt{2mr} \mp \sqrt{2rm\Delta}]^2 (3r - 2m)$$

$$= 2r^2m[r(r - m)(2m - r) + m\Delta]^2$$

Equation determines the radius of marginally stable circular orbits.

In case of extreme Kerr metric (a = m), gives for direct orbit (upper sign),

$$^{r}ms = m, \frac{3 + \sqrt{5}}{2} m$$

and for retrograde orbit (lower sign),

$$^{r}ms = 9m$$

Wilkins has illustrated by plotting L against r that all orbits with radius <– 5.3 are co-revolving (L>0). Hence this also shows that the orbit corresponding to rms = (3 +√5)m/2 is a direct orbit.

Observation and Calculation: Three segments of orbits are like idea of year, Month and day raised in the Vedic era as explained above and the ergo sphere of Kerr geometry is the region between the surface of stationary

$rt = m + (m^2 - aCos^2\theta)^{½}$ (i.e. where g00 = D = 0) and the "one-way membrane" or event horizon r0 = m + (m2 - a)½. A particle entering the ergo sphere can, if it is properly powered, escape again to infinity. The surface of stationary is sometimes called a surface of infinity red shift. Only in the case of the Schwarzschild geometry (a = 0) does the surface of stationary coincide with the event horizon. In the general Kerr geometry, the surfaces are separated everywhere except at the poles. In case of Kerr geometry, we have two surfaces "interior null surface", r = m - (m2-a2)½, the other is the "interior g00 = 0 surface", r = m - (m2 – aCos2θ)½. Neither of these surfaces can make itself felt to a far-way

observer. In the case of Schwarzschild geometry (a = 0), these two interior surfaces coalesce to two singularity, r = 0. In the equatorial plane, the surface of stationary takes the form rt = 2m. Hence the orbit -

$$^{r}ms = m, \frac{3 + \sqrt{5}}{} m$$

is the direct orbit lying outside the ergo sphere of the Kerr metric.Here, in case of Schwarzschild metric (a = 0) we get from

$$r_{ms} = 2m, 6m$$

As has been started that the maximum binding energy is given by

(1 - E/μ). At a smaller radius the value of E/μ becomes greater than one and the orbits become unbounded as well as being unstable. Hence the radius of marginally bound circular orbit can be obtained from the equation

$$\frac{E}{\mu} = 1$$

Equation together with give

$$K^2 = \frac{2mr}{r^2 + a^2}$$

$$\left[ar(r - m)\sqrt{2mr} \mp \sqrt{2rm\Delta}\right]2$$

$$= 2rm[r(r - m)(2m - r) + m\Delta]^2$$

Equation determines the radius of marginally bound orbits.

For extreme Kerr metric , gives for direct orbit (upper sign)

$$r_{mb} = m, \frac{3+\sqrt{5}}{2}\ m$$

and for retrograde orbit (lower sign)

$$r_{mb} = (3 + \sqrt{2})m$$

For Schwarzschild metric we get

$$r = 2m, 4m$$

An unstable circular photon orbit is obtained when both E/μ and L/μ become infinitely large. This will be the case when in the limit

$$2r^2 - k^2 (r^2 + a^2) \rightarrow 0$$

From previous result, we get the following expression for the radius of unstable photon circular orbit -

$$[ar(r - m)\underline{\sqrt{2mr}} \underline{+} \underline{\sqrt{2}}rm\Delta]^2$$

$$= 2r^2[r(r - m)(2m - r) + m\Delta]^2$$

In case of extreme Kerr metric we get from equation,

$$r_{ph} = m, \frac{3+\sqrt{5}}{2} m$$

r_{ph} = 4m for retrograde orbit

For Schwarzschild case (a = 0) we have

$$r_{ph} = 2m, 3m$$

Coming on the same fact ,It is obvious as has been found out by Bardeen,here also we get that the direct marginally stable orbit, the direct marginally bound orbit, the direct photon orbit and the event horizon seem to coincide at r = m where a = m. The apparent conflict with the null character of the event horizon is because the co-ordinate 'r' misrepresents the geometry of the space time near r = m where a = m.

By putting $r = m(1 + f)$, $a = m(1 - \delta)$, $\delta << 1$ and $f \to 0$ (with upper sign) and approximating up to the appropriate order of f and δ,we get for the three orbits near r = m the following expressions:

$$r_{ms} = m\,[1 + (4\delta)^{1/3}],$$

$$r_{mb} = m\,[1 + 2\delta^{1/2}],$$

$$r_{ph} = m\,[1 + \{(8/3)\delta\}^{1/2}],$$

These results are the same as have been obtained by Bardeen.

The limiting energies E and velocities $\upsilon\varphi$ as $a \to m$ for the orbits and are obtained from and as follows:

$$E/\mu = 1/\sqrt{3} = 0.58,\ \upsilon\varphi \to 1/2 \text{ at } r = r_{ms}$$

$$E/\mu = 1,\qquad \upsilon\varphi \to 2^{-1/2} \text{ at } r = r_{mb}$$

As $a \rightarrow 0$, the Kerr metric tends to become the Schwarzschild metric. If in the Kerr metric terms containing a2 and higher powers of a are neglected, it become :

$$ds2 = \frac{dr2}{\{1 - (2m/r)\}} + r^2 d\theta^2 + Sin^2 \theta d\varphi^2 - \{1 - (2m/r)\} dt^2$$

$$-(4ma/r) dtd\varphi Sin^2\theta$$

Here, equation spews that the metric represented by it has a small departure from spherical symmetry of Schwarzschild metric. This logic leads to suppose another path of analysis. We consider a Schwarzschild black hole. Notwithstanding spherical cloud of dust gets accreted into this hole. A spherical cloud of dust will collapse to Schwarzschild black hole provided only that it is not endowed with angular momentum. In case it has less than a critical angular momentum, it will settle down to a uniquely defined but deformed standard black hole configuration (Kerr geometry). This is the conclusion to which one has been led by the analysis presented by Israel and others. A full dynamical analysis by Price reveals that as time runs out all small perturbations in the metric of Schwarzschild goes to zero, but the angular momentum is conserved and its effect on the metric does not decay with time, neither does it lead to any singularity near the Schwarzschild surface. The small perturbation due to the angular momentum is given by,

$$h_{03} = \frac{-Sin2\theta(\text{angular momentum})}{r} = \frac{-2Sin2\theta.ma}{r}$$

This perturbation is the same as shown which is the limiting case of Kerr metric as and terms containing a2 and higher powers of a are neglected. We proceed to consider the stable circular orbits near the event horizon of such a perturbed metric.

For the Kerr metric the event horizon is given by,

$$r_t = m + \sqrt{m^2 - a^2}$$

putting $\alpha = a/m$, reduces to

$$r_t = 2m(1 - \alpha^2/4)$$

where higher powers of α higher than α^2 have been neglected. it is evident that the event horizon for the Kerr metric tends to become the event horizon rt = 2m of perturbed Schwarzschild metric if terms containing α^2 and higher powers of $\alpha 2$ are neglected. rt = 2m is also the event horizon of Schwarzschild metric.

We will consider here the nature of marginally stable, marginally bound and photon circular orbits near the event horizon of perturbed metric.

From above equations, it appears that in case a = 0(Schwarzschild metric), the orbits corresponding to $r = r_{ms}$, $r = r_{mb}$ and r = rph coincide the event horizon $r_t = 2m$.

Here In the case of the perturbed metric these circular orbits are obtained by putting r = 2m (1 +f) and making approximation by up to first order in f and α we get respectively for the three orbits:

$$r_{ms} = 2m[1 + \alpha/\sqrt{2}]$$

$$r_{ms} = 2m[1 + \alpha/\sqrt{2}]$$

$$r_{ph} = 2m[1 + \alpha/\sqrt{2}]$$

The limiting energies and velocities υφ for the orbit r = r_{ms} are obtained from and and for r = r_{ms}, They are obtained . They are given respectively as:

The perturbed Schwarzschild metric may also be considered to represent the gravitational field of slowly totaling body.

From the above equations, it is evident that there is a direct orbit r = (3 +√5)m/2 where the direct marginally stable, the marginally bound and the direct circular photon orbit coincide. That the orbit r = (3 +√5)m/2 lies outside the ergo sphere has already been stated above. It would be interesting to know the variation in these orbits with the variation in 'a/m'.

Substituting r = m(p +f), a = m(1 - δ) where δ <- 1, f→0 with p = (+ 1)/2, approximating up to first order in f and δ we get -

$$f = -\frac{4d}{\sqrt{5}(\sqrt{5}+1)},$$ in each case and the three orbits respectively are given by,

$$r_{ms} = m\left[\frac{3+\sqrt{5}}{2} - \frac{4\delta}{\sqrt{5}(\sqrt{5}+1)}\right]$$

$$r_{mb} = m\left[\frac{3+\sqrt{5}}{2} - \frac{4\delta}{\sqrt{5}(\sqrt{5}+1)}\right]$$

$$r_{ph} = m \left[\frac{3+\sqrt{5}}{2} - \frac{4\delta}{\sqrt{5}(\sqrt{5}+1)} \right]$$

As a→m, the three orbits tend to coincide with r = m[(3 +√5)/2]but always remaining outside the ergo sphere.

The limiting energies E and velocity υφ as a→m for the orbits are obtained respectively as follows:

$$\frac{E}{\mu} \rightarrow \frac{5^{1/4}}{\sqrt{3}} = .86,\ \upsilon\varphi \rightarrow .69 \text{ for } r = r_{ms}$$

$$E/\mu \rightarrow 1 \qquad \upsilon\varphi \rightarrow 1 \text{ for } r = r_{mb}$$

It appears that the three orbits coincide even after the first approximation in *f* and δ.

It has been noticed that the three orbits became different when an approximation is made by retaining third degree terms in *f* and δ. After such an approximation, we get two values for rms and rph. The three are given as follows.

$$r_{ms} = m \left[\frac{3+\sqrt{5}}{2} - \frac{4\delta}{\sqrt{5}(\sqrt{5}+1)} - \frac{2}{5}\frac{(8\sqrt{5}-17)}{\sqrt{5}(\sqrt{5}+1)}\delta^2 + p_1\delta^3 \right]$$

$$\simeq m\,[2.6 - 0.6\delta - 0.04\delta^2 - 1.5\delta^3]$$

$$r'_{ms} = m \left[\frac{3+\sqrt{5}}{2} - \frac{4\delta}{\sqrt{5}(\sqrt{5}+1)} - \frac{2}{5}\frac{(8\sqrt{5}-17)}{\sqrt{5}(\sqrt{5}+1)}\delta^2 + p_1\delta^3 \right]$$

$$\simeq m\,[2.6 - 0.66 - 0.04\delta^2 - 0.2\delta^3]$$

$$\text{where } p_1 = \frac{[4(216\sqrt{5} - 484) - 2\times5^{1/4}(\sqrt{5}+1)(271 - 115\sqrt{5})]}{25(5+\sqrt{5})(2 - 5^{1/4})}$$

$$\simeq 1.50$$

$$\text{where } p2 = \frac{[4(216\sqrt{5} - 484) - 2\times5^{1/4}(\sqrt{5} + 1)(271 - 115\sqrt{5})]}{25(5 + \sqrt{5})(2 - 5^{1/4})}$$

$$\simeq + 0.20$$

$$^{r}mb = m\left[\frac{3+\sqrt{5}}{2} - \frac{4\delta}{\sqrt{5}(\sqrt{5}+1)} - \frac{2}{5}\frac{(8\sqrt{5}-17)}{\sqrt{5}(\sqrt{5}+1)}\delta^2 + \delta^2 + \frac{4(108\sqrt{5}-242)}{25(5+\sqrt{5})}\delta^3\right]$$

$$\simeq m[2.6 - 0.6\delta - 0.04\delta^2 - 0.03\delta^3]$$

$$^{r}ph = m\left[\frac{3+\sqrt{5}}{2} - \frac{4\delta}{\sqrt{5}(\sqrt{5}+1)} - \frac{2}{5}\frac{(8\sqrt{5}-17)}{\sqrt{5}(\sqrt{5}+1)}\delta^2 + \delta^2 + q_1\delta^3\right]$$

$$\simeq m[2.6 - 0.6\delta - 0.04\delta^2 - 0.85\delta^3]$$

$$^{r'}ph = m\left[\frac{3+\sqrt{5}}{2} - \frac{4\delta}{\sqrt{5}(\sqrt{5}+1)} - \frac{2}{5}\frac{(8\sqrt{5}-17)}{\sqrt{5}(\sqrt{5}+1)}\delta^2 + \delta^2 + q_1\delta^3\right]$$

$$\sim- m[2.6 - 0.6\delta - 0.04\delta^2 - 0.4\delta^3]$$

$$\text{where } q_1 = \frac{2(41\sqrt{5}-43)}{125} \simeq - 0.85$$

$$q_2 = \frac{2(41\sqrt{5}-43)}{25(5+\sqrt{5})} \simeq - 0.40$$

The three orbits defer only in the coefficients of δ3. It appears that if a higher approximation is carried out, the difference in the three orbits may be more marked. A strong preference for the equatorial plane would require that 'a/m not be small compare with one and hence δ<<1. To maintain this, we are not to retain terms in δ than of degree higher than a reasonable limit

Facts from Ancient Indian Literature:

I Am Time, Says Krishna:

In an article, Mathuresa Dasa says: "In the Bhagavad-Gita, Lord Krishna, the Supreme Personality of Godhead, gives his own answer in a few words. 'Time I am. The great destroyer of the worlds.' Time, according to the Gita and other Vedic literatures, is an inconceivable energy of the Supreme Lord, through which He ultimately destroys everything." . "Beyond the walls of the Universe that we know, Time assumes a different feature". We measure time in terms of the movements of physical objects. The time the earth takes to orbit the sun, we name as a year. **The time the moon takes to orbit the earth, we name as a month. And the time the earth takes to revolve on its axis, we name as a day. To further divide our days into hours, minutes and seconds, we observe other objects.**

Courtesey: https://www.speakingtree.in/allslides/what-did-krishna-mean-when-he-said-i-am-time/174264

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